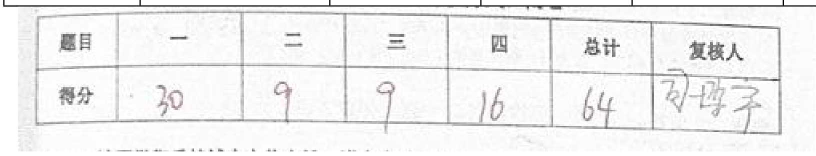
Academic Year 2023–2024, Semester 2

Biomedical Engineering Major, Class of 2021 / Biomedical Engineering (Upgraded Undergraduate), Class of 2023

"Digital Signal Processing" Exam Paper A

Duration: 90 minutes, Closed-book Exam

Question	一	二	三	四	Total	Revi
Score	30	9	9	16	64	Si Boyu



题目	一	二	三	四	总计	复核人
得分	30	9	9	16	64	[illegible]

Note: Students are expected to uphold the principles of honesty and integrity, abide by exam discipline, and refrain from cheating. Any violation of exam rules will be dealt with strictly in accordance with the "Shanghai University of Medicine and Health Sciences Student Disciplinary Regulations."

Score	Graded by
30	Wang Xuan

Part 1: Multiple Choice Questions (20 questions, 2 points each, 40 points total. Select the correct answer; no points will be awarded for unanswered or incorrect answers.)

Question	1	2	3	4	5	6	7	8	9	10
Option	C	A	A	D	B	A	A	D	C	B
Question	11	12	13	14	15	16	17	18	19	20
Option	B	D	C	B	D	B	B	C	A	B

得分	阅卷人
30	王[illegible]

一、单项选择题（共 20 题，每题 2 分，共 40 分。选择正确项得分，不选或选错误项均不得分）

-10

题目	1	2	3	4	5	6	7	8	9	10
选项	C	A	A	D	B	A	A	D	C	B
题目	11	12	13	14	15	16	17	18	19	20
选项	B	D	C	B	D	B	B	C	A	B

1.The sequences x(n)={0，1/2，1，3/2},h(n)={1，1，1}are given. The length of their convolution sum is（ ）

A.4　　B.5　　C.6　　D.7

2. $x(n) = A\cos(5\pi n/8 + \pi/6)$,The period of the sequence is（　　）

A.16　　B.16π　　C.5　　D.5π

3.If a continuous-time signal is ideally sampled and satisfies the Nyquist sampling theorem without spectral aliasing, it can be accurately reconstructed using a (　　)

试卷装订线

A.Ideal low-pass filter B.Ideal high-pass filter C.Ideal band-pass filter D.Ideal band-stop filter

4.Which of the following systems is linear？（ ）

A.$y(n) = Re[x(n)]$ B.$y(n) = kx(n) + m$

C.$y(n) = [x(n)]^2$ D.$y(n) = \sum_{m=-\infty}^{n} x(m)$

5. Which of the following linear time-invariant systems is non-causal？（ ）

A. $h(n) = \frac{1}{n!}u(n)$ B. $h(n) = -a^n u(-n-1)$

C. $h(n) = 0.5^n u(n)$ D. $h(n) = \delta(n)$

6.Generally, the region of convergence for a ( ) sequence is a circle with the radius equal to the smallest pole magnitude of X(z)

A.Left-sided B.Right-sided C.Two-sided D.Causal

7.$X(z)=1/(1-z^{-1})$,|z|>1， the x(n) is（ ）

A.u(n) B.u(-n) C.-u(n) D.-u(-n-1)

8. Z[x(n)]=X(z)， 1<|z|<2，the Z[x(-n)] is（ ）

A.X(-z),1<|z|<2 B.X(-z),0.5<|z|<1 C.X(1/z),1<|z|<2 D. X(1/z),0.5<|z|<1

9. The imaginary axis in the S-plane corresponds to the ( ) in the Z-plane（ ）

A. Imaginary axis B.Real axis C.Unit circle D.Ray

10. Given the signal x(n)={$\underline{4}$,3,2,1}if $x_1(n) = x((n-2))_4 R_4(n)$， then $x_1(n)$ is（ ）

A. $x_1(n)$={$\underline{1}$,2,3,4} B. $x_1(n)$={$\underline{2}$,1,4,3}

C. $x_1(n)$={$\underline{2}$,4,1,3} D. $x_1(n)$={$\underline{4}$,3,2,1}

11. The expression for the DFT of a sequence is $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$ ，From this, the length of the sequence in the time domain and the interval between two adjacent frequency points in the transformed frequency domain are ( )

A. $N, \frac{2\pi}{N}$ B. $N-1, \frac{2\pi}{N}$ C. $N, \frac{2\pi}{N+1}$ D. $N-1, \frac{2\pi}{N+1}$

12. Which of the following statements about the properties of the Discrete Fourier Transform (DFT) is incorrect?（ ）

A.The DFT is a linear transformation

B.The DFT has implicit periodicity

C.The DFT can be viewed as the sampling of the sequence's Z-transform on the unit circle

D.The DFT can precisely analyze the spectrum of continuous signals

13. Regarding the FFT, which of the following statements is incorrect?（ ）

A.The FFT is a new type of transform

B.The FFT is a fast algorithm for the DFT

C.The FFT can be divided into time-decimation and frequency-decimation methods

D.The base-2 FFT requires the sequence length to be a power of 2

14. For an N-point radix-2 FFT with $N=2^L$（where L is an integer）, the number of stages of butterfly operations required is （ ）

A. N　　B. L　　C.2L　　D.2N

15. Which of the following is not a commonly used indicator for characterizing filter performance in practice? ( )

A.Filter order　B.Maximum passband attenuation C.Minimum stopband attenuation　D.Passband cutoff frequency

16. For a Butterworth low-pass filter with order N=2, the number of poles in the magnitude-squared function is（ ）

A. 2　　B. 4　　C. 6　　D. 8

17. For a Butterworth low-pass filter with order N=4 时，, the angular spacing between the poles of $H_a(s)$ $H_a(-s)$ on the S-plane is（ ）

A. π/2　　B. π/4　　C. π/6　　D. π/8

18. If a Butterworth low-pass filter requires $\delta_p = 4dB$, then $\left|H\left(j\Omega_p\right)\right|$ is（ ）

A. 0.2　　B. 0.37　　C. 0.63　　D. 1

19. Which step in the design process of an IIR digital filter is incorrect? （ ）

A. Convert the given digital filter specifications into analog low-pass filter specifications

B. Design a normalized analog low-pass filter prototype $H_{an}(s)$ based on the converted specifications

C.Transform $H_a(s)$into $H_{an}(s)$

D. Convert the analog filter into a digital filter

20. Given the difference equation of a filter: $y(n) = -0.5y(n-1) + x(n)$, , its system function is

（ ）

A. $H(Z) = \dfrac{1}{1+0.5Z^{-1}}$　　B. $H(Z) = \dfrac{1}{1-0.5Z^{-1}}$

C. $H(Z) = \dfrac{Z^{-1}}{1+0.5Z^{-1}}$　　D. $H(Z) = \dfrac{Z^{-1}}{1-0.5Z^{-1}}$

Score	Graded by
9	Bai Baodan

Part 2: True or False (10 questions, 1 point each, 10 points total. Mark √ for correct statements and × for incorrect ones.)

1. Sampling a continuous signal in the time domain results in a periodic extension of its spectrum in the frequency domain. (√)

2.The system represented by y(n)=g(n)x(n) is time-invariant. (×)

3.The region of convergence for a right-sided sequence is the exterior of a circle with radius equal to the largest pole magnitude of X(z). (√)

4.The Z-transform on the unit circle is equivalent to the Laplace transform of the sequence. (×)

5.Increasing the data length N during spectrum analysis can improve physical resolution. Therefore, zero-padding at both ends of the signal can enhance physical resolution.

(×)

6.Spectral leakage and aliasing are inseparable. Leakage can cause aliasing because it spreads the spectrum, potentially allowing high-frequency components to exceed the folding frequency, leading to distortion. (√)

7.A digital filter is a discrete-time system that can selectively remove certain frequency components from the input signal to achieve signal processing. (√)

8.The poles of the Butterworth low-pass filter function Ha(s)Ha(-s) are distributed on an elliptical circumference in the S-plane. (×)

9.Mapping an analog filter to a digital filter involves transforming the Z-plane into the S-plane. (√)

10.An FIR filter with h(n)={0,1,3,1,2} has linear phase. (×)

得分	阅卷人
9	顾丹

二、判断题。正确的在括号内打√，错误的在括号内打×。

（共 10 题，每题 1 分，共 10 分）

1. 在时域对连续信号进行抽样，在频域中所得频谱是原信号频谱的周期延拓。 (√) √
2. y(n)=g(n)x(n) 所代表的系统是非移变系统。 (×) √
3. 右边序列的收敛域是以 X(z) 的最大极点模值为半径的圆外。 (√) √
4. 单位圆上的 Z 变换就是序列的拉普拉斯变换。 (×) √
5. 在进行频谱分析的时候，增加数据长度 N 可以增加频谱的物理分辨率，因此为了增加物理分辨率可以对信号两端进行补零操作。 (×) √
6. 频谱泄漏和频谱混叠是分不开的，泄漏会造成混叠，因为泄漏会导致频谱扩展，从而使得最高频率成分有可能超过折叠频率，从而造成混叠失真。 (√) √
7. 数字滤波器是一种离散时间系统，可以根据需要有选择性地滤除输入信号中的某些频率成分，从而实现对输入信号的处理。 (√) √
8. 巴特沃斯低通滤波器函数 Ha(s)Ha(-s) 的极点分布在 s 平面的椭圆圆周上。 (×) √
9. 模拟滤波器映射成数字滤波器，就是要寻找某种映射，将 Z 平面映射成 S 平面。 (√) × -1
10. 一个 FIR 滤波器的 h(n)={0,1,3,1,2}，则该滤波器具有线性相位。 (×) √

Score	Graded by
9	Li Xiaoou

Part 3: Fill-in-the-Blank Questions (15 questions, 2 points each, 30 points total)

得分	阅卷人
9	李晓欧

三、填空题（共 15 题，每题 2 分，共 30 分）

-21

1.According to the Nyquist sampling theorem, to avoid distortion when sampling a band-limited signal, the sampling frequency f_s and the signal's highest frequency f_{max} must satisfy the relationship: $\underline{\quad f_{\max} \geq 2 f_s \quad}$.

2.A linear system must satisfy both additivity and <u>decomposability</u>.

3.For a linear time-invariant system with impulse response $h(n) = a^n u(n)$,(where a is a real constant), the condition for system stability is $\underline{\quad 0 \leq \sum_{-\infty}^{\infty} a_n \leq \infty \quad}$.

4.The bilateral Z-transform of a discrete sequence x(n) is defined as X(z)__________________.

5.Methods for finding the inverse Z-transform include the contour integral method (residue theorem), partial fraction decomposition, and the power series method (long division).

6.If Z[x(n)]=X(z), then Z[x(n+m)]= $x(mz)$.

7.The Fourier transform of a sequence is defined as DTFT[x(n)]= $\sum_{n=0}^{N-1} x(n)e^{-j\omega n}$.

8.The N-point DFT of $\delta(n)$ is 1 .

1. 从奈奎斯特采样定理可知，要使一个带限信号采样后不失真，采样频率f_s与该信号的最高频率f_{max}的关系为 $f_{max} \geq 2f_s$. X -2
2. 线性系统需同时满足可加性和 可分解性 X -2
3. 设线性非移变系统的单位脉冲响应$h(n)=a^n u(n)$，a为实常数，该系统稳定的条件是 $0 \leq \sum a_n < \infty$. X -2
4. 一个离散序列 x(n) 的双边 Z 变换定义为 X(z)=__________. X -2
5. 求逆 z 变换的方法有围线积分法(留数法)、部分分式展开法和幂级数法(长除法)。
6. 设 Z[x(n)]=X(z)，则 Z[x(n+m)]= X(mz) . X -2
7. 序列的傅里叶变换定义为 DTFT[x(n)]= $\sum_{n=0} x(n)e^{-jwn}$ X -1
8. 计算$\delta(n)$的 N 点 DFT 为 $X(k)=\sum_{n=0} x(n)W_N^{nk}=1$

9.A continuous periodic time-domain signal is necessarily aperiodic discrete in the frequency domain.

10.The basic computational unit of the radix-2 FFT algorithm is the____________________.

11.The magnitude response of a Butterworth filter depends on N. As the filter order N increase, the passband becomes flatter, and the transition band becomes sharper.

12.Common methods for mapping analog filters to digital filters include frequency response invariant method, the step-invariant method, and the bilinear transform method.

13.Physically realizable filters typically include a ______ between the passband and stopband.

14.FIR filters can achieve strict low frequency characteristics while allowing arbitrary magnitude responses.

15.In the basic IIR structures, direct Form II requires fewer section than direct Form I.

9. 一个连续周期的时域信号，其在频域必然是 非周期离散.
10. 基-2 FFT 算法的基本单元为________. X -2
11. 巴特沃斯滤波器幅度响应与 N 有关，随着滤波器阶次 N 的 增加 ，通带越平坦，过渡带越陡峭。
12. 常用的将模拟滤波器映射成数字滤波器的方法有 频率响应~~模~~ 法、阶跃响应不变法和双线性变换法。 -2
13. 物理上可实现的滤波器一般在通带和阻带之间设置一个__________. X -2
14. FIR 滤波器可以在幅度特性随意设计的同时，保证严格的 低频 特性。X -2
15. 在 IIR 基本结构中，直接 II 型比直接 I 型的所用的 环节 少。X -2

Score	Graded by

Part 4: Short Answer Questions (4 questions, 5 points each, 20 points total)

得分	阅卷人
16	[illegible]

四、简答题（共4题，每题5分，共20分）

-4

1.Given the sequence x(n)=sin(nπ/4)+sin(nπ/6)， determine its period.

1.令序列 x(n)=sin(nπ/4)+sin(nπ/6)，求 x(n)的周期。

5

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\frac{\pi}{4}} = 8$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

取T_1, T_2的最小公倍数24

∴ x(n)的周期为24

Take the least common multiple of T1 and T2 as 24.

Thus, the period of x(n) is 24.

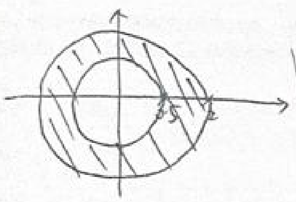
2. For the sequence $x(n)=0.5^{|n|}$, find its Z-transform and region of convergence.

5

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

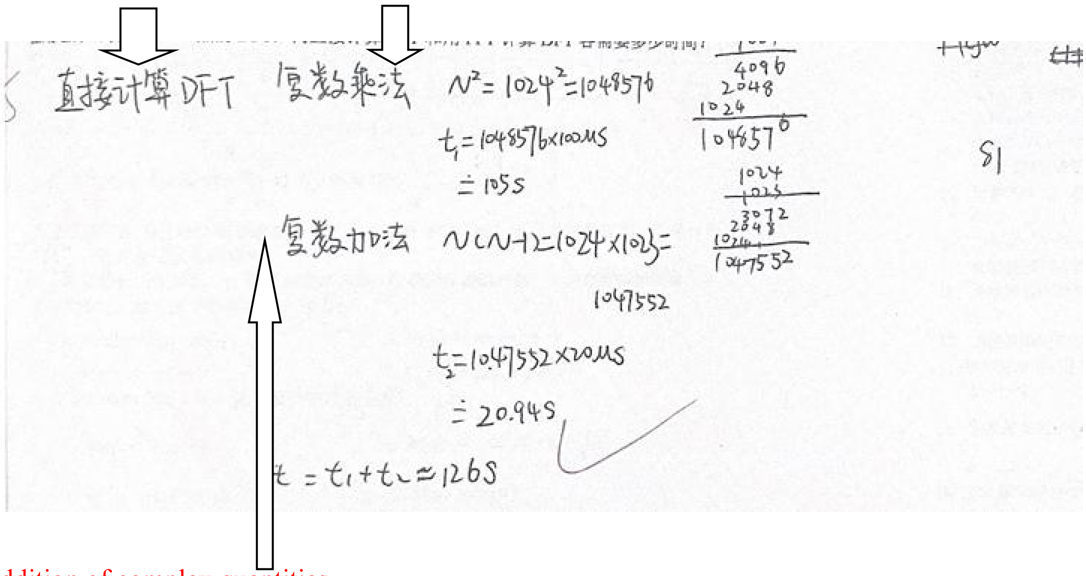
$$= \sum_{-\infty}^{-1} 0.5^{-n} z^{-n} + \sum_{0}^{+\infty} 0.5^{n} z^{-n}$$

$$= \frac{0.5z}{1-0.5z} + \frac{1}{1-0.5z^{-1}}$$



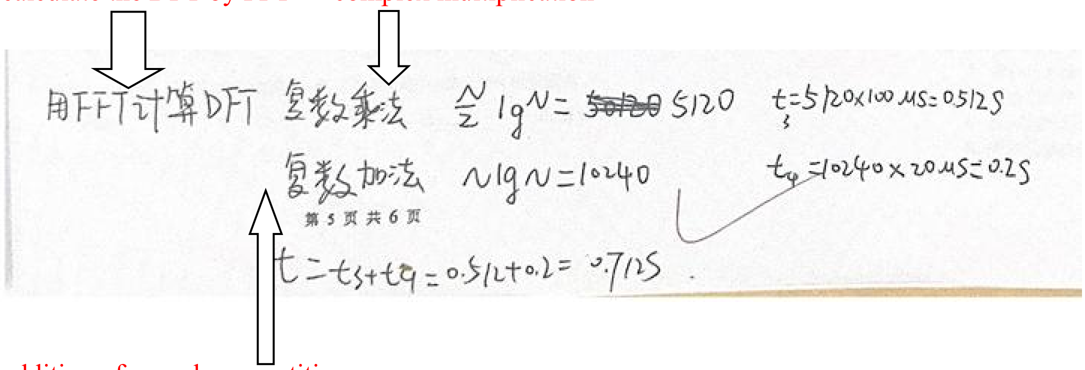
3. If a general-purpose computer takes 100 μs to compute one complex multiplication and 20 μs for one complex addition, calculate the time required to compute a 1024-point DFT directly and using the FFT.

Directly calculate the DFT complex multiplication



addition of complex quantities

calculate the DFT by FFT complex multiplication



addition of complex quantities

4. Design a Butterworth filter meeting the following specifications: passband edge frequency $\Omega_p = 15\pi\ rad/s$, maximum passband attenuation $\delta_p = 3dB$, stopband edge frequency $\Omega_{st} = 25\pi\ rad/s$, and minimum stopband attenuation $\delta_{st} = 20dB$. The coefficients of the normalized filter system function are provided in Table.

Table 1: Coefficients of the Normalized Filter System Function

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	1								

2	1.4142								
3	2	2							
4	2.6131	3.4142	2.6131						
5	3.2361	5.2361	5.2361	3.2361					
6	3.8637	7.4641	9.1416	7.4641	3.8637				
7	4.494	10.0978	14.5918	14.5918	10.0978	4.4940			
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1371	5.1258		
9	5.7588	16.5817	31.1634	41.9864	41.9864	16.5817	31.1634	5.7588	
10	6.3925	20.4317	42.8021	64.8824	74.2334	64.8824	42.8021	20.4317	6.3925

| 10 | 6.3925 | 20.4317 | 42.8021 | 64.8824 | 74.2334 | 64.8824 | 42.8021 | 20.4317 | 6.3925 |

$\cancel{H(j\omega}$ $\cancel{\frac{}{1+(\frac{\Omega}{\Omega_c})^{2N}}}$ $|H_a(j\omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$

8| $\frac{10^{\delta_p/10}-1}{10^{\delta_{st}/10}-1} = \frac{\Omega_p}{\Omega_{st}}.$

−4